SYNCHRONOUS MACHINES. THEORY

FUNDAMENTALS OF CONSTRUCTION AND OPERATION

1. 3-phase symmetrical winding at the stator (another name – armature); u, v, w.

2. DC winding at the rotor (field winding, exciting winding) supplied with DC field current $I_f$ to produce a rotor magnetic field of definite polarity.

3. Rotor flux is sinusoidally distributed in space (along the air-gap periphery).

   - $d$ – direct axis of rotor
   - $q$ – quadrature axis of rotor

Sinusoidal distribution of the rotor flux density can be realised in two main ways:

a) in cylindrical synchronous machines (typical construction of turbogenerators)

   ![Diagram of cylindrical synchronous machines]

   $B(x) \equiv B_m \cos \left( \frac{x}{\tau_p} \right)$

b) in salient-pole synchronous machines (typical construction of hydrogenerators)

   ![Diagram of salient-pole synchronous machines]

   It is easy to prove by means of magnetic calculation that for appropriately sized air-gap thickness the flux density distribution can be achieved as required. In our case:

   $\delta(x) = \frac{\delta_o}{\cos \left( \frac{x}{\tau_p} \right)}$  \hspace{1cm} $B(x) \equiv B_m \cos \left( \frac{x}{\tau_p} \right)$

The cross-section of the rotor body is cylindrical. The field winding is of the form of distributed winding (not concentrated coil)

Turbogenerator – popular synchronous generator of electrical energy in thermal power stations

Hydrogenerator – synchronous generator driven by water turbine in hydro power station.

At the rotor we have now two (or more) outstanding magnetic poles. Field winding has the form of concentrated coil. In such arrangement, when the air-gap thickness between stator and rotor shoe was constant the flux density distribution would be rectangular.
4. The rotor is driven and the sinusoidally distributed flux rotates at rotational (angular) speed $\omega$. In effect there are electromotive forces induced in stator windings:

\[ e_u = \sqrt{2}E_f \sin \omega t \quad E_f - \text{rms value of emf in stator winding due to rotor flux} \]

\[ e_v = \sqrt{2}E_f \sin \left( \omega t - \frac{2}{3} \pi \right) \]

\[ e_w = \sqrt{2}E_f \sin \left( \omega t - \frac{4}{3} \pi \right) \]

\[ E_f = 4.44f_{kw} \Phi_f \]

\[ f_t = \frac{p n_1}{60} \quad f_1 = \frac{\omega}{2 \pi} \]

5. In the closed circuit of stator windings the currents flow due to emfs:

\[ i_u = \sqrt{2}I \sin (\omega t - \Psi) \]

\[ i_v = \sqrt{2}I \sin \left( \omega t - \frac{2}{3} \pi - \Psi \right) \]

\[ i_w = \sqrt{2}I \sin \left( \omega t - \frac{4}{3} \pi - \Psi \right) \]

Such 3-phase symmetrical currents flowing in 3-phase symmetrical stator windings produce rotating magnetic field of stator – sinusoidally distributed in space. Direction of rotation – accordingly to phase sequence, i.e. in the same direction as rotor rotates. The field is called armature reaction field. Its rotational speed $n_1 = \frac{60f_1}{p}$ is of the same value as rotor speed.

**Summary:** rotor, rotor field and stator (armature) field rotate synchronously (with the same speed $n_1$) ⇒ SYNCHRONOUS MACHINE!

**PHASOR REPRESENTATION**

Armature reaction flux induces emf $E_a$

\[ E_a = \alpha \Phi_a = cI \quad E_a = -jX_a I \]

or

\[ U_a = jX_a I \] – voltage drop at the reactance $X_a$

$X_a$ – armature reaction reactance (reactance of the stator winding corresponding to the armature reaction flux).

$E$ – total emf induced by total (resultant) flux $\Phi$. It is also called the air-gap emf (as it corresponds to resultant air-gap flux). 

$\Psi$ - angle of displacement between emf $E_f$ and current $I$ in stator winding.

Symmetry of load is assumed.

In normal steady-state operation everything rotate synchronously.
SPECIAL CASES OF ARMATURE REACTION

a) direct-axis armature reaction

For such conditions of $\Phi_a$ the stator winding is characterised by:

$X_{ad}$ – direct-axis armature reaction reactance.

$\Psi = \pi/2$ corresponds to pure inductive load current (with respect to $E_f$). Consider also $\Psi = -\pi/2$ (capacitive character of load).

b) quadrature-axis armature reaction

$\Psi = 0$ – active load current.

For such conditions of $\Phi_a$ the stator winding is characterised by:

$X_{aq}$ – quadrature-axis armature reaction reactance.

For cylindrical machines: $X_{ad} \approx X_{aq}$
For salient-pole machines $X_{ad} > X_{aq}$

When we take into consideration also the leakage flux of stator winding, each phase will be characterised by the sum of reactances:

$X_l + X_{ad} = X_d$ – direct-axis synchronous reactance,

$X_l + X_{aq} = X_q$ – quadrature-axis synchronous reactance.

For cylindrical machines: $X_d \approx X_q$
For salient-pole machines $X_d > X_q$

Check carefully and compare the magnetic conditions for fluxes $\Phi_a$ in both cases: d and q positions in space and time.
c) Any other position of \( I \) and \( \Phi_a \)

- for cylindrical machine – magnetic path for \( \Phi_a \) remains the same and \( X_d = X_a \) can be assumed as parameter representing the stator winding;

- for salient-pole machine – magnetic path for \( \Phi_a \) is variable and depends on \( \Psi \). For each value of \( \Psi \) an appropriate value of \( X_a \) should be determined or superposition method of \( d \)-armature reaction and \( q \)-armature reaction should be applied (Blondel’s diagram).

**EQUIVALENT CIRCUIT OF CYLINDRICAL MACHINE (GENERATOR)**

\[ \begin{align*}
E_f & = U + jX_{ad}I + jX_lI + R_I I \\
E_f & = U + jX_{al}I
\end{align*} \]

\( U_s = (X_{ad} + X_l)I = X_aI \) - voltage drop at synchronous reactance

Voltage balance equations: