PERMANENT MAGNET AND SWITCHED RELUCTANCE MOTORS

INTRODUCTION

Electric machines can be broadly classified into two categories on the basis of how they produce torque - electromagnetically or by variable reluctance. In the first category, motion is produced by the interaction of two magnetic fields, one generated by the stator and the other by the rotor. Two magnetic fields, mutually coupled, produce an electromagnetic torque tending to bring the fields into alignment. The same phenomenon causes opposite poles of bar magnets to attract and like poles to repel. The vast majority of motors in commercial use today operate on this principle. These motors, which include DC and induction motors, are differentiated based on their geometries and how the magnetic fields are generated. Some of the familiar ways of generating these fields are through energized windings, with permanent magnets, and through induced electrical currents.

In the second category, motion is produced as a result of the variable reluctance in the air gap between the rotor and the stator. When a stator winding is energized, producing a single magnetic field, reluctance torque is produced by the tendency of the rotor to move to its minimum reluctance position. This phenomenon is analogous to the force that attracts iron or steel to permanent magnets. In those cases, reluctance is minimized when the magnet and metal come into physical contact. As far as motors that operate on this principle, the switched reluctance motor (SRM) falls into this class of machines.

1. SWITCHED RELUCTANCE MOTOR (SRM)

In construction, the SRM is the simplest of all electrical machines. Only the stator has windings. The rotor contains no conductors or permanent magnets. It consists simply of steel laminations stacked onto a shaft. It is because of this simple mechanical construction that SRMs carry the promise of low cost, which in turn has motivated a large amount of research on SRMs in the last decade.

The mechanical simplicity of the device, however, comes with some limitations. Like the brushless DC motor, SRMs can not run directly from a DC bus or an AC line, but must always be electronically commutated. Also, the saliency of the stator and rotor (SRMs are doubly salient singly excited electric motors - both rotor and stator have salient poles), necessary for the machine to produce reluctance torque, causes strong non-linear magnetic characteristics, complicating the analysis and control of the SRM (SRM torque expression is derived from first principles and requires a relationship between machine flux linkages or inductance and rotor position). Their concentrated coil phases are turned - on sequentially, to produce torque, through d.c. voltage pulses which result in unipolar controlled current.

Not surprisingly, industry acceptance of SRMs has been slow. This is due to a combination of perceived difficulties with the SRM, the lack of commercially available electronics with which to operate them, and the entrenchment of traditional AC and DC machines in the marketplace.

SRMs do, however, offer some advantages along with potential low cost. For example, they can be very reliable machines since each phase of the SRM is largely independent
physically, magnetically, and electrically from the other motor phases. Also, because of the lack of conductors or magnets on the rotor, very high speeds can be achieved, relative to comparable motors.

Power ratings of SRMs, ranges from a few watt to practically MW units for low speed control range low dynamics, as well as high grade (servo) applications, especially in thermally and chemically harsh environments.

1.1. CONSTRUCTION AND FUNCTIONAL ASPECTS

The basic operating principle of the SRM is quite simple; as current is passed through one of the stator windings, torque is generated by the tendency of the rotor to align with the excited stator pole. The direction of torque generated is a function of the rotor position with respect to the energized phase, and is independent of the direction of current flow through the phase winding. Continuous torque can be produced by intelligently synchronizing each phase’s excitation with the rotor position. By varying the number of phases m, the number of stator poles N_s (N_s = 2mq - each phase is made of concentrated coils placed on 2q stator poles), and the number of rotor poles N_r, many different SRM geometries can be realized. A few examples are shown in figure 1.0.

Generally, increasing the number of SRM phases reduces the torque ripple, but at the expense of requiring more electronics with which to operate the SRM. At least two phases are required to guarantee starting, and at least three phases are required to insure the starting direction. The number of rotor poles and stator poles must also differ to insure starting.

Most favored configurations - amongst many more options - are the 6 / 4 three phase and the 8 / 6 four phase SRMs (figure 1.1.a, b).

These two configurations correspond to q = 1 (one pair of stator poles (and coils) per phase) but q may be equal to 2, 3 when, for the three phase machine, we obtain 12 / 8 or 18 / 12 topologies applied either for low speed high torque direct drives or for high speed stator -
generator systems for aircraft. The stator and rotor pole angles $\beta_s$ and $\beta_r$ are in general almost equal to each other to avoid zero torque zones.

The symmetry of magnetic circuit leads to the almost zero mutual flux linkage in the SRM phases even under saturated conditions. This means that the SRM may work with $m-1$ phases since no induced voltage or current will appear in the short-circuited phase. Hence the SRM is more fault tolerant than any a.c. motor where the interaction between phases is at the core of their principle of operation. The self-inductance of each phase alone thus plays the key role in torque production.

Figure 1.1. Representative SRM configurations

In the absence of magnetic saturation, the phase self-inductance varies linearly with rotor position, while, in presence of saturation, the respective dependence is non-linear (figure 1.2).

Figure 1.2. The phase inductances and the operation modes of three phase 6 / 4 SRM.

If the phase flux linkage $\lambda$ is calculated and plotted versus current for various rotor positions, the $\lambda(\theta_r,i)$ curve family is obtained (figure 1.3). The influence of magnetic saturation is evident from figure 1.3 and it is a practical reality in well designed SRMs.
Phase coil has \( N \) turns, and when it is excited with current \( I \), the coil sets up a flux \( \Phi \). Increasing the excitation current will make the rotor pole (armature) move towards the stator pole. The flux vs. magnetomotive force (mmf) is plotted for two values of rotor position \( x_1 \) and \( x_2 \) as shown in figure 1.4. The flux vs. mmf characteristic for \( x_1 \) is linear because the reluctance of the air gap is dominant, making the flux smaller in the magnetic circuit.

The electrical input energy is written as:

\[
W_e = \int e \, dt = \int i \, dt \, \frac{dN \Phi}{dt} = \int N i \, d\Phi = \int F \, d\Phi,
\]

where \( e \) is the induced emf and \( F \) is the mmf. Input electrical energy, \( W_e \) is equal to the sum of the energy stored in the coil \( W_f \) and energy converted into mechanical work \( W_m \):

\[
W_e = W_f + W_m.
\]

When no mechanical work is done (start from the position \( x_1 \)), the stored field energy is equal to the input electrical energy given by (1.1). This corresponds to area OBEO in figure 1.4. The complement of the field energy, termed coenergy is given by area OBAO. And
mathematically expressed as $\int \Phi dF$. Similarly for the position $x_2$ of the rotor the field energy corresponds to area OCDO and coenergy is given by area OCAO. For incremental changes equation (1.2) is written as:

$$\delta W_e = \delta W_f + \delta W_m.$$  \hspace{1cm} (1.3)

For a constant excitation of $F_1$ given by the operating point $A$ in figure 1.4, the various energies are derived as:

$$\delta W_e = \int_{\phi_1}^{\phi_2} F_1 d\phi = F_1 (\phi_2 - \phi_1) = \text{area}(BCDEB),$$  \hspace{1cm} (1.4)

$$\delta W_f = \delta W_{f|x=x_2} - \delta W_{f|x=x_1} = \text{area}(OCDO) - \text{area}(OBEO).$$  \hspace{1cm} (1.5)

Using equations (1.3) to (1.5), the incremental mechanical energy is derived as:

$$\delta W_m = \delta W_e - \delta W_f = \text{area}(OBCO),$$  \hspace{1cm} (1.6)

and that is the area between the two curves for given magnetomotive force. In the case of a rotating machine, the incremental mechanical energy in terms of the electromagnetic torque and change in rotor position is written as:

$$\delta W_m = T_e \delta \theta,$$  \hspace{1cm} (1.7)

where $T_e$ is the electromagnetic torque and $\delta \theta$ is the incremental rotor angle. Hence the electromagnetic torque is given by:

$$T_e = \frac{\delta W_m}{\delta \theta}. \hspace{1cm} (1.8)$$

For the case of constant excitation (mmf is constant), the incremental mechanical work done is equal to the rate of change of coenergy $W'_f$, which is nothing but the complement of the field energy. Hence the incremental mechanical work done is written as:

$$\delta W_m = \delta W'_f,$$  \hspace{1cm} (1.9)

where:

$$W'_f = \int \Phi dF = \int \Phi d(Ni) = \int (N\Phi \delta \Phi) \int \lambda(\theta, i) \, di = \int L(\theta(i)) \, di,$$

$$\delta W_m = \frac{\delta W_m}{\delta \theta} = \frac{\delta W'_f}{\delta \theta} = \frac{\delta W'_f(i, \theta)}{\delta \theta} \bigg|_{i = \text{constant}}.$$  \hspace{1cm} (1.10)

where the inductance $L$ and flux linkages $\lambda$ are functions of rotor position and current. This change in coenergy occurs between two rotor positions $\theta_2 (x_2)$ and $\theta_1 (x_1)$. Hence, the air gap torque in terms of coenergy represented as a function of rotor position and current is:

$$T_e = \frac{\delta W_m}{\delta \theta} = \frac{\delta W'_f}{\delta \theta} = \frac{\delta W'_f(i, \theta)}{\delta \theta} \bigg|_{i = \text{constant}}.$$  \hspace{1cm} (1.11)
If the inductance is linearly varying with rotor position for given current, which in general is not the case in practice, then the torque can be derived as:

\[ T_e = \frac{dL(0\theta_i) i^2}{d\theta} \frac{d\theta}{2}, \quad (1.12) \]

where:

\[ \frac{dL(0\theta)}{d\theta} = \frac{L(\theta_2, i) - L(\theta_1, i)}{\theta_2 - \theta_1} \mid i = \text{constant}, \quad (1.13) \]

and this differential inductance can be considered to be the torque constant expressed in Nm/A². In fact it is not constant and it varies continuously. This has the implication that the SRM will not have a steady-state equivalent circuit in the sense that the DC and AC motors have. Equation (1.12) has the following implications:

1) The torque is proportional to the square of the current, hence the current can be unipolar to produce unidirectional torque. Only one power switch per phase winding is required.
2) The torque constant is given by the slope of the inductance vs. rotor position characteristic (SRM will not have a steady-state equivalent circuit in the sense that the DC and AC).
3) Since the torque is proportional to the square of the current, this machine resembles DC series motor and has a good starting torque.
4) Direction of rotation can be reversed by changing the sequence of stator excitation.
5) Torque and speed control is achieved with converter control.
6) SRM required controllable converter.
7) Mutual coupling between phases is practically absent. Short-circuit fault in one phase winding has no effect on the other phases (application in aircrafts, nuclear power plant coolant pumps).
8) One power switch per phase winding implies no shooting-through failure.

**1.2. EQUIVALENT CIRCUIT**

The mathematical model of SRM is highly non-linear due to magnetic saturation influence on the \( \lambda(\theta, i) \) curve family but it allows for phase - torque superposition as the interaction between phases is minimal.

As SRM has doubly saliency, stator (phase) coordinates are mandatory. The phase equations are:

\[ V_{a,b,c,d} = r_{a,b,c,d} + \frac{d\lambda_{a,b,c,d}(\theta_1, i_{a,b,c,d})}{dt}, \quad (1.14) \]

with the family of curves \( \lambda_{a,b,c,d}(\theta_1, i_{a,b,c,d}) \) obtained for one phase only (as the periodicity is \( \pi/N_\theta \)). These curves may be obtained either through theory or through tests. Analytical or finite element methods are used for the scope. Accounting for magnetic saturation and air gap flux fringing is mandatory in all cases.

The motion equations (neglecting bearing loses and friction) are:
Let us use the subscript $i$ for dominating one phase. Equation (1.14) may be written as:

$$
V_i = r_i i_i + \frac{\partial \lambda_i}{\partial i_i} \frac{di_i}{dt} + \frac{\partial \lambda_i}{\partial \theta_r} \frac{d\theta_r}{dt}.
$$  

(1.17)

Denoting $\frac{\partial \lambda_i}{\partial i_i}$ as the transient inductance $L_i$:

$$
L_i(\theta_r, i_i) = \frac{\partial \lambda_i(\theta_r, i_i)}{\partial i_i}.
$$  

(1.18)

The last term in (1.17) represents the back e.m.f. $E_i$:

$$
E_i = \frac{\partial \lambda_i}{\partial \theta_r} \cdot \omega_r.
$$  

(1.19)

So (1.17) becomes:

$$
V_i = r_i i_i + L_i(\theta_r, i_i) \frac{di_i}{dt} + E_i(\omega_r, \theta_r, i_i).
$$  

(1.20)

An equivalent circuit with time dependent parameters may be defined based on equations (1.20) - (figure 1.5).

Figure 1.5. Equivalent circuit of SRM with core losses

The core losses are represented by the variable resistances in parallel with the e.m.f. $E_i$, based on the assumption that only the main flux produces core losses. Core losses occur both in the stator and in the rotor core as this machine does not operate on the traveling field principle.

Especially in high speed applications (above 6000 rpm) core loss has to be considered not only for efficiency calculations but also in the transient current responses assessment.

Only for the linear case (no magnetic saturation) the instantaneous torque $T_d(t)$ is:
As usually heavy magnetic saturation is present, \( E_t(\omega_r, \theta_r, i_i) \) as of (1.19) is a pseudo e.m.f. as it includes also a part related to the magnetic energy storage. Consequently the torque is to be calculated only from the coenergy formula.

### 1.3 CONTROL

Sumarising, the instantaneous torque \( T_e(i) \) per phase may be obtained through the known energy, \( W_{mc}(\theta) \), formula:

\[
T_e(i) = \frac{\partial W_{mc}(\theta)}{\partial \theta} \bigg|_{\theta_{osc}} \cdot \int_0^i \lambda(\theta_r,i) \, di
\]

Equation (1.22) demonstrates the necessity of knowing, through calculations or test, the family of curves \( \lambda(\theta_r,i) \). The total instantaneous torque is (m – number of phases):

\[
T_e = \sum_{i=1}^{m} T_e(i)
\]

Only in the absence of saturation the instantaneous torque is:

\[
T_e = \sum_{i=1}^{m} \frac{1}{2} i_i \frac{\partial \lambda(\theta_r,i)}{\partial \theta_r}
\]

SRM drives are controlled by synchronizing the energization of the motor phases with the rotor position. Figure 1.6 illustrates the basic strategy.
As equation (18) suggests, positive (or motoring) torque is produced when the motor inductance is rising as the shaft angle is increasing, dL/dθ>0. Thus, the desired operation is to have current in the SRM winding during this period of time. Similarly, a negative (or braking) torque is produced by supplying the SRM winding with current while dL/dθ<0. The exact choice of the turn-on and turn-off angles and the magnitude of the phase current, determine the ultimate performance of the SRM. The design of commutation angles, sometimes called firing angles, usually involves the resolution of two conflicting concerns - maximizing the torque output of the motor or maximizing the efficiency of the motor. In general, efficiency is optimized by minimizing the dwell angle (the dwell angle is the angle traversed while the phase conducts), and maximum torque is achieved by maximizing the dwell angle to take advantage of all potential torque output from a given phase.

Ideally a phase is turned - on when rotor poles, along the direction of motion, lay between neighbouring stator poles θ₀ = 0 (figure 1.7), for the motoring operation mode of the respective phase. Only one voltage pulse is applied for a conduction (dwell) angle θₜₚ = θₜₚ₀ - θₜₜ₀ in figure 1.7.

![Figure 1.7. Phase inductance, voltage, flux linkage and current](image)

During this period, neglecting the resistive voltage drop, the maximum phase flux linkage λₘₐₓ, for constant speed ωᵣ, is:

\[
\lambda_{\text{max}} = \int_{\theta} V_d \, dt = V_d \frac{\theta_w}{\omega_r} \quad (1.25)
\]

\[
\omega_r = V_d \frac{\theta_w}{\lambda_{\text{max}}}
\]

The maximum value of θₜₚ, for θₜₜ₀ = 0 (zero advance angle), is given by motor design:
The base speed $\omega_b$ corresponds to $\theta_{w_{\text{max}}}$ and single voltage pulse $V_d$ with maximum flux linkage $\lambda_{\text{max}}$, which is dependent on machine design and the level of saturation. Thus, to reach higher speeds $\omega_r$ (above $\omega_b$), we have to saturate the magnetic circuit of that machine (Equation 1.25). This is called flux weakening.

The smaller the angle $\theta_{\text{off}} - \theta_m$, the smaller the negative torque "contribution" of the phase going off. In reality at $\theta_r = \theta_m$ (aligned position) if the current $i_m$ is already less than (25 - 30%) of the peak current, the negative (generating) torque influence becomes small.

Once one phase is turned off at $\theta_r$ another one is turned on, eventually soon, to contribute positive torque such that to lower the total torque pulsations caused by the reduction of torque in the phase going off.

It is now evident that the entire magnetic energy of each phase is "pumped" in and out for each conduction cycle. There are $mN_r$ cycles per mechanical revolution. A part of this energy is passed over to the incoming phase through the power electronic converter (P.E.C.) and the rest to the d.c. bus filtering capacitor of P.E.C. Below base speed $\omega_b$ the current is limited (and controlled) through PWM (figure 1.8).

It should be noticed that the interval of conduction is prolonged close to $\theta_m$ where the phase inductance is maximum. As mentioned above, at high speeds the phase turn - on angle $\theta_{\text{on}}$ is advanced and so is the turn - off angle $\theta_c$.

SRM control is often described in terms of "low-speed" and "high-speed" regimes. Low-speed operation is typically characterized by the ability to arbitrarily control the current to any desired value. Figure 1.8 illustrates waveforms typical of low-speed SRM operation. As the motor’s speed increases, it becomes increasingly difficult to regulate the current because of a combination of the back EMF effects and a reduced amount of time for the commutation interval. Eventually a speed is reached where the phase conducts (remains on) during the entire commutation interval. This mode of operation, depicted by Figure 1.9, is called the single-pulse mode.
When this occurs, the motor speed can be increased by increasing the conduction period (a greater dwell angle) or by advancing the firing angles, or by a combination of both. Example of experimental change of RRSRM (rolling Rotor Switched Reluctance Motor) speed by using both above technique is presented in figure 1.10.

By adjusting the turn-on and turn-off angles so that the phase commutation begins sooner, we gain the advantage of producing current in the winding while the inductance is low (the current reaches its maximum - at $\theta_{mc}$, sooner and at a higher level and thus produce more torque), and also of having additional time to reduce the current in the winding before the rotor reaches the negative torque region (the turn-off process of a phase starts at $\theta_c \leq \theta_m$ and terminates at $\theta_{off}$ in the "generating" zone). Control of the firing angles can be accomplished a number of ways, and is based on the type of position feedback available and the optimization goal of the control. When position information is more precisely known, a more sophisticated approach can be used. One approach is to continuously vary the turn-on angle with a fixed dwell.
Near turn-on, equation (1.22) can be approximated as

\[ V_i = \frac{\partial \lambda_i}{\partial i} \frac{di}{dt} = L_i \frac{di}{dt}. \]  

(1.27)

Multiplying each side of equation (1.27) by the differential, d\( \theta \), and solving for d\( \theta \), gives:

\[ d\theta = \frac{L_i}{V_i} \frac{d\theta}{dt}, \]  

(1.28)

and using first order approximations yields an equation for calculating advance angle:

\[ \theta_{\text{adv}} = \frac{L_i i_i}{V_{\text{bus}}} \omega, \]  

(1.30)

where \( i_i \) is the desired phase current and \( V_{\text{bus}} \) is the DC bus voltage.

### 1.4. CONNECTING AND COMMUTATION PROCESS

Phase coil connecting process is shown in figure 1.11.

![Fig.1.11. Connecting process.](image)

General law that governs this circuit is as follow:

\[ U = Ri + L \frac{di}{dt}. \]  

(1.31)

To solve above equation one must find transient current and stationary current (Fig.1.13). Stationary current satisfies equation:

\[ \frac{di_s}{dt} = 0, \]  

(1.32)
where: \( i_w = \frac{U}{R} \). \hfill (1.33)

Transient current satisfies equation:

\[ Ri_s + L \frac{di_s}{dt} = 0, \] \hfill (1.34)

which solution is as follow:

\[ i_s = Ae^{-\frac{R}{L}t}, \] \hfill (1.35)

where constant A one can find from boundary conditions: \( i(0)=i_w(0)+i_s(0) \Rightarrow i_s(0)=-\frac{U}{R}=A \). So:

\[ i = i_w + i_s = \frac{U}{R} (1 - e^{-\frac{R}{L}t}), \] \hfill (1.36)

were: \( R/L \) – time constant (snubbing coefficient).

Disconnecting process is shown in Fig.1.12.

![Diagram of disconnecting process](image)

Fig.1.12. Disconnecting process.

To avoid electric arc circuit inductance must be short circuited. General law that governs this process is as follow:

\[ Ri_s + L \frac{di_s}{dt} = 0, \] \hfill (1.37)

and then:

\[ U_L = -U_R = L \frac{di_s}{dt} = -Ue^{-\frac{R}{L}t}, \] \hfill (1.38)

where: \( \tau = L/R \).
Equivalent electronic circuit is shown in figure 1.15a, where commutation diode allows creating short circuit for inductance current after "switch off" the main power transistor. Winding coil deenergizing process is slow and depends on circuit time constant $\tau = \frac{L}{R}$. The instantaneous value of RRSRM phase current is presented in figure 1.14.

To avoid breaking torque production due to existing the current during angle $\theta_{off} - \theta_m$ some techniques are used to shorten deenergizing process. One possibility is to reverse voltage during switch off process using the unipolar current bridge per phase (figure 1.14b).
Unipolar current bridge offers 3 independent conduction modes. This circuit provides maximum control flexibility and efficiency with minimum active components. If transistor K1 is “on” and K2 is “on” the output load terminal is short-circuited to supply and voltage across the load is +Uz (figure 1.16a.). If K2 was formerly conducted and switches “off”, the current commutates to diode D1, and the inductance load keeps the current flowing (figure 1.16b). This is equivalent to one key switch operation. If K1 and K2 are both “off” the voltage across the load reverses polarity from +Uz to –Uz (equivalent of applying reverse voltage as it is shown in Figure 1.9.), and remains at this value until the current decays to zero (figure 1.16c.).
2. PERMANENT MAGNET BRUSHLESS MOTOR (BL DC)

A typical topology of a conventional d.c. brush motor with stator PM (or electromagnetic) excitation and a rotor comprising the armature winding and the mechanical commutator with brushes is shown in figure 2.1a. The mechanical commutator is in fact an electromechanical DC - AC bi-directional power flow power converter as the currents in the rotor armature coils are a.c. while the brush - current is DC. Brushes make mechanical contact with a set of electrical contacts on the rotor (called the commutator), forming an electrical circuit between the DC electrical source and the armature coil-windings. As the armature rotates on axis, the stationary brushes come into contact with different sections of the rotating commutator. The commutator and brush system form a set of electrical switches, each firing in sequence, such that electrical-power always flows through the armature coil closest to the stationary stator pole (permanent magnet or electrically excited).

Figure 2.1. a) Conventional DC brush motor with cylindrical rotor.  
b) PM DC motor with disk rotor.

Figure 2.1b shows an axial air gap PM DC brush motor with a printed - winding ironless - disk rotor and the mechanical commutator with brushes. PM excitation, especially
with the nonmagnetic disk rotor, yields extremely low electric time constants L/R (around or less than 1 ms in the sub kW power range). Thus quick response in current (torque) is expected, though the current (torque) harmonics are large unless the switching frequency in the PEC is not high enough.

Unfortunately the mechanical commutator though not bad in terms of losses and power density has serious commutation current and speed limits and thus limits the power per unit to 1 - 2 MW at 1000 rpm and may not be at all accepted in chemically aggressive or explosion - prone environments.

In a BLDC motor, the electromagnets do not move; instead, the permanent magnets rotate and the armature remains static (figure 2.2).

![Fig.2.2. Crosssection of BL DC motor](image)

This gets around the problem of how to transfer current to a moving armature. In order to do this, the brush-system/commutator assembly is replaced by an intelligent electronic controller. The controller performs the same power distribution found in a brushed DC motor, but using a solid-state circuit rather than a commutator/brush system.

BLDC motors offer several advantages over brushed DC motors, including higher reliability, reduced noise, longer lifetime (no brush erosion), elimination of ionizing sparks from the commutator, and overall reduction of electromagnetic interference (EMI). The maximum power that can be applied to a BLDC motor is exceptionally high, limited almost exclusively by heat, which can damage the magnets. BLDC’s main disadvantage is higher cost, which arises from two issues. First, BLDC motors require complex electronic speed controllers to run. Brushed DC motors can be regulated by a comparatively trivial variable resistor (potentiometer or rheostat), which is inefficient but also satisfactory for cost-sensitive applications. Second, many practical uses have not been well developed in the commercial sector.

BLDC motors are considered to be more efficient than brushed DC motors. This means that for the same input power, a BLDC motor will convert more electrical power into mechanical power than a brushed motor, mostly due to the absence of friction of brushes. The enhanced efficiency is greatest in the no-load and low-load region of the motor's performance curve. Under high mechanical loads, BLDC motors and high-quality brushed motors are comparable in efficiency.
2.1. IDEAL BRUSHLESS DC MOTOR WAVEFORMS

Let us suppose that the PM produces a rectangular air gap flux distribution over \( \alpha_{PM} = \pi (180^0) \) (figure 2.3.a). Generally the surface PM extends over an angle \( \alpha_{PM} \) less than \( 180^0 \) (which represent the pole pitch, figure 2.2). The stator phase mmf is supposed to be rectangular, a case corresponding to \( q = 1 \) (three slots per pole). Consequently the PM flux linkage in the stator winding \( \lambda_{PM}(\theta_{er}) \) varies linearly with rotor position (figure 2.3.b). Finally the phase emf \( E_a \) is rectangular with respect to rotor position (figure 2.3.c)

\[
\lambda_{PM}(\theta_{er}) = \left( 1 - \frac{2}{\pi} \theta_{\alpha} \right) \lambda_{PM}
\]  

(2.1)

The maximum flux linkage (for \( W_1 \) turns and \( B_g \) air gap flux density) per phase \( \lambda_{PM} \) is:
\[ \lambda_{PM} = W_r E_{PM} r L. \]  \hspace{1cm} (2.2)

The phase emf, \( E_a \), is:

\[ E_a = -\frac{d\lambda_{PM}(\theta_a)}{d\theta_a} \frac{d\theta_a}{dt} = \frac{2}{\pi} \lambda_{PM} \omega_r \]  \hspace{1cm} (2.3)

where \( \omega_r \) is the electrical angular speed.

As two phases conduct at any time the ideal torque \( T_e \) is constant:

\[ T_e = 2E_a i_{dc} \frac{p}{\omega_r} = \frac{4}{\pi} \lambda_{PM} o_i i_{dc}. \]  \hspace{1cm} (2.4)

Between any two current instantaneous commutation the phase current is constant (\( i_d = i_{dc} \)) and thus the voltage equation is:

\[ V_d = 2r_s i_{dc} + 2E_a = 2r_s i_{dc} + \frac{4}{\pi} \lambda_{PM} o_i. \]  \hspace{1cm} (2.5)

From (2.5) and (2.5) we obtain:

\[ \omega_r = \omega_{r0} \left(1 - \frac{T_e}{T_{esc}} \right) \]  \hspace{1cm} (2.6)

\[ \omega_{r0} = \frac{\pi}{4} \frac{V_d}{\lambda_{PM}}; \ T_{esc} = \frac{4}{\pi} \lambda_{PM} p i_{sc}; \ i_{sc} = \frac{V_d}{2r_s} \]  \hspace{1cm} (2.7)

The ideal speed - torque curve is linear (figure 2.4) like for a d.c. brush PM motor.

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![Figure 2.4. Ideal speed - torque curves of BLDC](image-url)
The speed may be reduced and reversed by reducing the level and changing the polarity of d.c. voltage supplying each motor phase by proper commutation in the PWM inverter (figure 2.4). The current level is reduced by chopping.

The value of maximum flux linkage in a phase ($\lambda_{PM}$) may be reduced by advancing the currents in the phase by the angle $\alpha_a \neq 0$. On the other hand for an advancing angle $\alpha_a = \pi$ the electromagnetic power becomes negative and thus regenerative braking mode is obtained (figure 2.3.e).

2.2. CURRENT CONTROL SYSTEM

In general a rectangular current control system contains the BLDC motor, the PWM inverter, the speed and current controllers and the position (speed) sensors (or estimators, for sensorless control) and the current sensors (figure 11.7).

![Rectangular current control of BLDC](image)

Figure 2.5. Rectangular current control of BLDC

The currents sequence, produced through inverter adequate control, with $120^\circ$ current waveforms in figure 2.6 shows also the position of the 6 elements of the proximity sensors with respect to the axis of the phase a for a zero advance angle $\alpha_a = 0$.

The location of proximity sensors P(a+, a-, b+, b-, c+, c-) is situated $90^\circ$ (electrical) behind the pertinent phase with respect to the direction of motion. With two phases conducting the stator active mmf is on from $60^\circ$ to $120^\circ$ with respect to the rotor position. The ideal voltage vector (figure 2.6) also jumps $60^\circ$ for any phase commutation in the inverter. Each phase is on $120^\circ$ out of $180^\circ$ for the $120^\circ$ conducting strategy.

To reverse the speed the addresses (IGBTs) of the proximity sensor elements action are shifted by $180^\circ$ (P(a+) $\rightarrow$ P(a-); P(b+) $\rightarrow$ P(b-); P(c+) $\rightarrow$ P(c-)). The proximity sensor has been located for zero advance angle to provide similar performance for direct and reverse motion. However, through electronic means, the advance angle may be increased as speed increases to reduce the peak PM flux in the stator phase and thus produce more torque, for limited voltage, at high speeds.
Figure 2.6. a.) Current sequencing. b.) phase connection

Using the same hardware we may also provide for 180° conduction conditions, at high speeds, when all three phases conduct at any time.

The three phase full-bridge circuit can operate with Wye-connected phase winding (figure 2.7.) as well as with delta-connected phase winding (figure 2.8.).

Fig.2.7. Three phase bridge circuit for square wave drives. Wye connected motor.

Fig.2.8. Three phase bridge circuit for square wave drives. Delta connected motor.
In figure 2.9.) line current for three-phase bridge including the state of the transistors and current paths is presented.

![Diagram showing three-phase bridge operation]

In presented above operation it is seen that there is only one DC current which is switched or commutated among the phases. This imply that current could be measured with only one current sensor in the DC supply and regulated by chopping only one transistor. However, the operation of the circuit is complicated by the action of the freewheel diodes and the motor back-emf. Because of the diodes the three phase currents are not necessarily “observable” to a current sensor in the DC supply. For full control of the current all the times, usually current is measured in three lines. However the use of current sensor in the lines does not necessarily guarantee the detection of overcurrents as it is shown in figure 2.10.

![Diagram showing the three-phase bridge circuit showing conducting loops just after Q5 has turned off and Q1 has turned on. This is the start of the 60° “base interval”]

Fig. 2.9. Three-phase bridge operation

Fig. 2.10. Three-phase bridge circuit showing conducting loops just after Q5 has turned off and Q1 has turned on. This is the start of the 60° “base interval”.
2.3. THE HYSTERESIS CURRENT CONTROLLER

As it is stated above the DC current control requires only one current sensor, placed in the DC link, to regulate the current level. The distribution of the current through the six groups of two phases at a time is triggered by the proximity position sensor.

Quite a few current controllers may be applied for the scope. Discussion of the hysteresis controller allows a quick understanding of motor - inverter interactions [2]. Once the current in a phase is initiated (as triggered by the proximity sensor) it increases until it reaches the adopted maximum value $i_{\text{max}}$. In that moment the phase is turned off until the current decreases to $i_{\text{min}}$. The duration of on and off times, $t_{\text{on}}$ and $t_{\text{off}}$, is determined based on the hysteresis band: $2(i_{\text{max}} - i_{\text{min}})$ (figure 2.11).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure211.png}
\caption{Current chopping}
\end{figure}

Let us consider the motor equations for the on ($t_{\text{on}}$) and off ($t_{\text{off}}$) intervals with Q1 and Q4 and D1D4 in conduction (figure 2.12), that is a+b- conduction.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure212.png}
\caption{Conduction of phases a and b. a.) on – time. b.) off - time}
\end{figure}

During the on - time, figure 2.12.a, the a+b- equation is:

\begin{align}
V_d &= 2r_i i + 2L_s \frac{di}{dt} + E_a - E_b \\
E_a - E_b &\approx 2E_s = E_0 = \text{const.}
\end{align}

(2.8) 

(2.9)
If the turning on is advanced by $\alpha_a$ part of the on time $E_a - E_b = 0$ and thus a faster current increase is possible.

The solution of equation (2.8) is:

$$i(t) = \frac{V_d - E_0}{2\tau_s} \left( 1 - e^{-\frac{t}{\tau_s}} \right) + i_{m,k} e^{-\frac{t}{\tau_c}}$$  \hspace{1cm} (2.10)

To allow for current rising $V_d > E_0$. Above a certain speed $V_d < E_0$ and thus current chopping is not feasible any more. The current waveform contains in this case a single on-off pulse triggered by the proximity sensor (estimator).

During the off time (diodes D1 and D4 conducting, in figure 2.12.b) the voltage equation is:

$$0 = 2r_s i + 2L_s \frac{di}{dt} + (E_s - E_b) + \frac{1}{C_f} \int i \, dt + V_c$$  \hspace{1cm} (2.11)

$V_{c0}$ is the capacitor voltage at the end of on time (11.15), or at the beginning of off-time. The solution of (2.11) with $t' = t - t_{on}$ is:

$$i(t) = -\frac{V_{c0}}{2\omega L_s} e^{-\alpha_1 t} \sin \omega t' - \frac{(E_s - E_b)}{\omega} e^{-\alpha_1 t} \sin (\omega t' - \phi)$$  \hspace{1cm} (2.12)

$$\omega_0 = \sqrt{\frac{1}{2 C_f L_s}}; \quad \alpha_1 = \frac{\tau_s}{2 L_s}; \quad \omega = \sqrt{\omega_0^2 - \alpha_1^2}; \quad \phi = \tan^{-1} \frac{\omega}{\alpha_1}$$  \hspace{1cm} (2.13)

The torque $T_e(t)$ expression is:

$$T_e = \frac{(E_s - E_b) i(t)}{\omega_{fr}}$$  \hspace{1cm} (2.14)

So if the e.m.f. is constant in time the electromagnetic torque reproduces the current pulsations between $i_{min}$ and $i_{max}$.

Exemplary instantaneous torque (2.14) includes current pulsations (as from (2.10) and (2.12)) for BLCD motor of the following parameters:

1 - motor is fed through a PWM inverter from a 300V d.c. source ($V_d = 300V$).
2 - rectangular current control is performed at an electrical speed $\omega_r = 2\pi 10$rad/s.
3 - no load line voltage at $\omega_r$ is $E_0 = 48V = const.$
4 - the cyclic inductance $L_s = 0.5mH$, $r_s = 0.1\Omega$ and the stator winding has $q = 1$ slot per pole per phase and two poles ($2p = 2$).
5 - the filter capacitor $C_f = 10mF$, the current chopping frequency $f_c = 1.25kHz$ and $t_{on}/t_{off} = 5/3$. 

is presented in figure 2.13.
It should be noticed that the chopping frequency is low for the chosen speed ($E_0 << V_d$) and thus the current and torque pulsations are large. Increasing the chopping frequency will reduce these pulsations. To keep the current error band $2(I_{max} - I_{min})$ within reasonable limits the chopping frequency should vary with speed (higher at lower speeds and lower at medium speeds) Though the high frequency torque pulsations due to current chopping are not followed by the motor speed, due to the much larger mechanical time constant, they produce flux density pulsations and, thus, notable additional core and copper losses (only the average current $I_0$ is, in fact, useful).

2.4. PRACTICAL PERFORMANCE

So far the phase commutation transients - current overlapping - have been neglected. They however introduce notable torque pulsations at $6\omega_r$ frequency (figure 2.14) much lower than those due to current chopping. To account for them complete simulation or testing is required [3].

There are also some spikes in the conducting phase when the other two phases commute (points A and B on figure 2.14).

Also not seen from figures (2.14 - 2.15) is the cogging torque produced at zero current by the slot - openings in presence of rotor PMs. Special measures are required to reduce the cogging torque to less than 2 - 5% of rated torque for high performance drives.
While at low speeds current chopping is feasible at high speeds one current pulse remains (figure 2.15). The current controller gets saturated and the required current is not reached.

As the advance angle is zero ($\alpha_a = 0$) there is a delay in "installing" the current and thus, as the emf is "in phase" with the reference current, a further reduction in torque occurs.

### 2.5. EXTENDING THE TORQUE - SPEED DOMAIN

Extending the torque - speed domain may be obtained (for a given drive) by advancing the phase commutation time by an angle $\alpha_a$ dependent on speed. This phase advancing allows fast current rise before the "occurrence" of the emf (assuming a PM span angle $\alpha_{PM} < 150^\circ - 160^\circ$). An approximate way to estimate the advance angle required $\alpha_a$, for $120^\circ$ conduction, may be based on linear current rise [4] to the value $I$:

\[
(\alpha_a)_{120^\circ} = \omega_r \frac{I}{V_{dc}}; \quad \omega_r = 2pn\pi
\]  

(2.23)

where $n$ is the rotor speed in rps.

Torque at even higher speeds may be obtained by switching from $120^\circ$ to $180^\circ$ current conduction (three phase working at any time). The current waveform changes, especially with advancing angle (figure 2.16).

This time the e.m.f. is considered trapezoidal, that is close to reality. The advancing angle $\alpha_a$ may be, for high speeds, calculated assuming sinusoidal emf [4] and current variation:

![Figure 2.15. Current waveform at high speeds](image)

![Figure 2.16. 180° conducting with advancing angle at high speed](image)
It has been demonstrated \[3,4\] that 120° conduction is profitable at low to base speeds while 180° conduction with advancing angle is profitable for high speeds (figure 2.17).

A smooth transition between 120° and 180° conduction is required to fully exploit the torque - speed capabilities of brushless DC motor drives.
3. PERMANENT MAGNET AND RELUCTANCE STEPPER MOTORS

A stepper motor converts electrical pulses into specific rotational movements. The movement created by each pulse is precise and repeatable, which is why stepper motors are so effective for positioning applications.

Stepper motors come in two varieties, permanent magnet and variable reluctance (there are also hybrid motors, which are indistinguishable from permanent magnet motors from the controller's point of view). When no power is applied permanent magnet motors tend to "cog" while rotor is revolved, while variable reluctance motors almost spin freely (although they may cog slightly because of residual magnetization in the rotor). Variable reluctance motors usually have three (sometimes four) windings, with a common return, while permanent magnet motors usually have two independent windings, with or without center taps. Center-tapped windings are used in unipolar permanent magnet motors.

Stepping motors come in a wide range of angular resolution. The coarsest motors typically turn 90 degrees per step, while high resolution permanent magnet motors are commonly able to handle 1.8 or even 0.72 degrees per step. With an appropriate controller, most permanent magnet and hybrid motors can be run in half-steps, and some controllers can handle smaller fractional steps or microsteps.

For both permanent magnet and variable reluctance stepping motors, if just one winding of the motor is energised, the rotor (under no load) will snap to a fixed angle and then hold that angle until the torque exceeds the holding torque of the motor, at which point, the rotor will turn, trying to hold at each successive equilibrium point.

3.1 VARIABLE RELUCTANCE MOTORS

Typical connection of variable reluctance stepping motor with three windings and with one terminal common to all windings is shown in the schematic diagram in figure 3.1. In use, the common wire typically goes to the positive supply and the windings are energized in sequence.

![Fig.3.1. Variable reluctance stepper motor](image)

The cross section shown in figure 3.1 is of 30 degree per step variable reluctance motor. The rotor in this motor has 4 teeth and the stator has 6 poles (that resemble SRM most common construction), with each winding wrapped around two opposite poles. With winding number 1 energised, the rotor teeth alligned with Y coordinate are attracted to this winding's poles. If the current through winding 1 is turned off and winding 2 is turned on, the rotor will rotate 30 degrees clockwise so that the poles alligned with X coordinate line up with the poles marked 2.
3.2 VARIABLE RELUCTANCE MOTORS OPERATION

To rotate this motor continuously, we just apply power to the 3 windings in sequence. Assuming positive logic, where a 1 means turning on the current through a motor winding, the following control sequence will spin the motor illustrated in figure 3.1 clockwise 24 steps or 2 revolutions:

<table>
<thead>
<tr>
<th>CW rotation</th>
<th>Bipolar step</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>CCW rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There are also variable reluctance stepping motors with 4 and 5 windings, requiring 5 or 6 wires. The motor geometry illustrated in Figure 1.1, giving 30 degrees per step, uses the fewest number of rotor teeth and stator poles that performs satisfactorily. Using more motor poles and more rotor teeth allows construction of motors with smaller step angle. Toothed faces on each pole and a correspondingly finely toothed rotor allows for step angles as small as a few degrees.

3.4 PERMANENT MAGNET STEPPER MOTOR

Permanent Magnet stepper motors incorporate a permanent magnet rotor, coil windings and magnetically conductive stators. Energizing a coil winding creates an electromagnetic field with a north and south pole as shown in figure 3.2.

![Magnetic field creating by energizing a coil winding.](image)

The stator carries the magnetic field which causes the rotor to align itself with the magnetic field. The magnetic field can be altered by sequentially energizing or “stepping” the stator coils which generates rotary motion. The motor cross section shown in figure 3.3 is of a 30 degree per step permanent magnet or hybrid motor -- the difference between these two motor types is not relevant at this level of abstraction.

![Permanent magnet stepper motor crosssection.](image)
Motor winding number 1 is distributed between the top and bottom stator pole, while motor winding number 2 is distributed between the left and right motor poles. The rotor is a permanent magnet with 6 poles, 3 south and 3 north, arranged around its circumference.

For higher angular resolutions, the rotor must have proportionally more poles. The 30 degree per step motor in the figure is one of the most common permanent magnet motor designs, although 15 and 7.5 degree per step motors are widely available. Permanent magnet motors with resolutions as good as 1.8 degrees per step are made. A rotor from a 7.5° motor has 12 pole pairs and each pole plate has 12 teeth. There are two pole plates per coil and two coils per motor; hence 48 poles in a 7.5° per step motor. Figure 3.4 illustrates the 4 pole plates of a 7.5° motor in a cut away view. Multiple steps can be combined to provide larger movements. For example, six steps of a 7.5° stepper motor would deliver a 45° movement.

Hybrid stepper motors are routinely built with 3.6 and 1.8 degrees per step, with resolutions as fine as 0.72 degrees per step available.

### 3.5 PERMANENT MAGNET STEPPER MOTOR OPERATION

Figure 3.5 illustrates a typical step sequence for a two phase motor shown in figure 3.3, called “one phase on” stepping.

![Fig.3.5. „One phase on” stepping sequence for two phase motor](image-url)
In Step 1 phase A of a two phase stator is energized. This magnetically locks the rotor in the position shown, since unlike poles attract. When phase A is turned off and phase B is turned on, the rotor rotates 90° clockwise. In Step 3, phase B is turned off and phase A is turned on but with the polarity reversed from Step 1. This causes another 90° rotation. In Step 4, phase A is turned off and phase B is turned on, with polarity reversed from Step 2. Repeating this sequence causes the rotor to rotate clockwise in 90° steps.

A more common method of stepping is “two phase on” where both phases of the motor are always energized. However, only the polarity of one phase is switched at a time, as shown in figure 3.6. With two phase on stepping the rotor aligns itself between the “average” north and “average” south magnetic poles. Since both phases are always on, this method gives 41.4% more torque than “one phase on” stepping, but with twice the power input.

The motor can also be “half stepped” by inserting an off state between transitioning phases. This cuts a stepper’s full step angle in half. For example, a 90° stepping motor would move 45° on each half step, figure 4. However, half stepping typically results in a 15% - 30% loss of torque depending on step rate when compared to the “two phase on” stepping sequence. Since one of the windings is not energized during each alternating half step there is less electromagnetic force exerted on the rotor resulting in a net loss of torque.
3.6 BIPOLAR WINDINGS

Bipolar coil windings are the most popular ones, are used in all variable reluctance motors and in most permanent magnet motors. Each phase consists of a single winding. If necessary by reversing the current in the windings, electromagnetic polarity is reversed. Thus, the motor itself is simpler but the drive is more complex. The output stage of a typical two phase bipolar drive, way of wiring and motor cross section is illustrated in figure 3.8.

![Diagram of Bipolar Winding](image)

The drive circuitry for such a motor requires an *H-bridge* control circuit for each winding. An H-bridge allows the polarity of the power applied to each end of each winding to be controlled independently. The control sequences for single stepping such a motor are shown in Table.1.

<table>
<thead>
<tr>
<th>CW rotation</th>
<th>Bipolar step</th>
<th>Q2-Q3</th>
<th>Q1-Q4</th>
<th>Q6-Q7</th>
<th>Q5-Q8</th>
<th>CCW rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Some permanent magnet stepping motors have 4 independent windings, organized as two sets of two (so called bifilar windings). Within each set, if the two windings are wired in series, the result can be used as a high voltage bipolar motor. If they are wired in parallel, the result can be used as a low voltage bipolar motor. If they are wired in series with a center tap, the result can be used as a low voltage unipolar motor.

3.7 UNIPOLAR WINDINGS

Another common winding is the unipolar winding. This consists of two windings on a pole connected in such a way (with a center tap on each of winding) that when one part of winding is energized a magnetic north pole is created, when the other is energized a south pole is created. This is referred to as a unipolar winding because the electrical polarity, i.e. current flow, from the drive to the coils is never reversed. This design allows for a simpler electronic drive. In use, the center taps of the windings are typically wired to the positive supply, and the two ends of each winding are alternately. However, there is approximately 30% less torque available compared to a bipolar winding. Torque is lower because the
energized coil only utilizes half as much copper as compared to a bipolar coil. The output stage of a typical two phase unipolar drive, way of wiring and motor cross section is illustrated in figure 3.9.

![Fig.3.9. Typical two phase unipolar drive.]

The stepping sequence is illustrated in Table.2.

<table>
<thead>
<tr>
<th>CW rotation</th>
<th>Bipolar step</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>CCW rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These sequences are identical to those for a bipolar permanent magnet motor, at an abstract level, and that above the level of the power switching electronics (for bipolar windings the H-bridge is used, for unipolar single key system is used), the control systems for the two types of motor can be identical.

### 3.8 MULTIPHASE WINDINGS

A less common class of permanent magnet or hybrid stepping motor is wired with all windings of the motor in a cyclic series, with one tap between each pair of windings in the cycle, or with only one end of each motor winding exposed while the other ends of each winding are tied together to an inaccessible internal connection. In the context of 3-phase motors, these configurations would be described as Delta and Y configurations, but they are also used with 5-phase motors, as illustrated in figure 1.5.

![Figure 3.10]
Some multiphase motors expose all ends of all motor windings, leaving it to the user to decide between the Delta and Y configurations, or alternatively, allowing each winding to be driven independently.

Control of either one of these multiphase motors in either the Delta or Y configuration requires 1/2 of an H-bridge for each motor terminal. It is noteworthy that 5-phase motors have the potential of delivering more torque from a given package size because all or all but one of the motor windings are energised at every point in the drive cycle. Some 5-phase motors have high resolutions on the order of 0.72 degrees per step (500 steps per revolution).

### 3.9 LINEAR ACTUATORS

The rotary motion of a stepper motor can be converted into linear motion by several mechanical means. These include rack & pinion, belt and pulleys and other mechanical linkages. All of these options require various external mechanical components. The most effective way to accomplish this conversion is within the motor itself. The linear actuator was first introduced in 1968.

Conversion of rotary to linear motion inside a linear actuator is accomplished through a threaded nut and leadscrew. The inside of the rotor is threaded and the shaft is replaced by a lead screw. In order to generate linear motion the lead screw must be prevented from rotating. As the rotor turns the internal threads engage the lead screw resulting in linear motion. Changing the direction of rotation reverses the direction of linear motion. The basic construction of a linear actuator is illustrated in figure 3.11.

![Figure 3.11. Linear actuator cut away showing threaded rotor to leadscrew interface.](image)

The linear travel per step of the motor is determined by the motor’s rotary step angle and the thread pitch of the rotor nut and leadscrew combination. Coarse thread pitches give...
larger travel per step than fine pitch screws. However, for a given step rate, fine pitch screws deliver greater thrust. Fine pitch screws usually can not be manually “backdriven” or translated when the motor is unenergized, whereas many coarse screws can. A small amount of clearance must exist between the rotor and screw threads to provide freedom of movement for efficient operation. This result in .001” to .003” of axial play (also called backlash). If extreme positioning accuracy is required, backlash can be compensated for by always approaching the final position from the same direction. Accomplishing the conversion of rotary to linear motion inside the rotor greatly simplifies the process of delivering linear motion for many applications. Because the linear actuator is self contained, the requirements for external components such as belts and pulleys are greatly reduced or eliminated. Fewer components make the design process easier, reduce overall system cost and size and improve product reliability.

3.10 STEPPER MOTOR DYNAMICS

Stepper motor is an electromechanical actuator that responds to a command in electrical input by displacing its rotor by a fixed angular displacement. Since it is controlled by electronic drive circuitry therefore it can be employed where there is a need to convert digital input pulses into analog shaft-output motion. Stepper motor should make fast response to an input pulse or pulse sequence. Both quick start and quick stop is required. If the pulse train is interrupted while the motor is running the motor should be capable of stopping at the position specified by the last pulse. Generally the highest the ratio of torque to rotor inertia the better is dynamic behavior of the motor. Step motor performance is well defined by torque vs. speed characteristics, oscillatory response to a single step and step length error. Therefore the laboratory stand for testing stepper motor should enable to measure, evaluate and draw these parameters and characteristics

3.10.1 OSCILLATORY RESPONSE TO A SINGLE STEP

There are several stepper motor constructions classified as permanent magnet, reluctance and hybrid type machines. Despite of the type difference stepper motor motion equation is as follow:

\[ J \frac{d^2 \Theta}{dt^2} + D \frac{d\Theta}{dt} + M_t \text{sign} \frac{d\Theta}{dt} + M_0 = m_e \] (3.1)

where:
- \( J \) - moment of inertia
- \( D \) - fluid friction coefficient
- \( M_t \) - moment of dry friction
- \( M_0 \) - load
- \( m_e \) - torque produced

Torque produced by stepper motor is the first derivative from magnetic energy \( W \) accumulated in motor in respect to the rotor angular position:

\[ m_e = \frac{\partial W}{\partial \Theta} = \frac{1}{2} \sum i \frac{\partial^2 y}{\partial \Theta} \] (3.2)
were: \( \Psi \) - magnetic flux linkage with phase winding
\( i \) - phase current

Primary cause of rotor mechanical oscillations is far quicker change of electromagnetic flux than rotor angular position change. In this case the surplus of energy that is introduced into the motor effects in mechanical oscillations. The smaller are electromagnetic time constants and internal motor dumping the greater are oscillations.

Kinetic energy accumulated in rotating mass depends on the angular rotor position:

\[
W_k = \int_0^\alpha_z M(\alpha_z - \alpha) d\alpha
\]  

(3.3)

As a result the rotor after reaching the align position \( (\alpha_z) \) of its tooth with energized pole, continues its movement in the area of reverse synchronizing torque. Rotor slows down to stop and in this position \( (\alpha_{max}) \) kinetic energy is equal to field energy:

\[
W_p = \int_{\alpha_z}^{\alpha_{max}} M(\alpha_z - \alpha) d\alpha
\]  

(3.4)

and electrical and mechanical losses:

\[
W_l = \int_{\alpha_{min}}^{\alpha_z} 2M(\alpha_z - \alpha) d\alpha
\]  

(3.5)

Then the rotor continues his movement backward. Oscillations around the align position decay due to dissipation of surplus energy (figure 3.1.).

Fig. 3.1. Energy dissipation and shaft oscillations around the align position.
Oscillatory response to a single step can be described by logarithmic dumping decrement:

$$\delta = \log\left(\frac{A_n}{A_{n+1}}\right)$$  \hspace{1cm} (3.6)

dumping oscillation frequency:

$$\omega = \frac{2\pi}{T}$$  \hspace{1cm} (3.7)

and free oscillation frequency:

$$\omega_f = \omega\sqrt{\frac{1 + \delta^2}{\pi^2}}$$  \hspace{1cm} (3.8)

were:  
$A_n$ - peak value of first cycle,
$A_{n+1}$ - peak value of second cycle,
$T$ - time between first and second peak.

The most difficult from the point of view of motor control is the case when the input pulses frequency is equal to the motor free oscillation frequency.

3.10.2 STATIC STEP ACCURACY

The angular position accuracy of a stepper motor varies from one step to the next. This inaccuracy is influenced by the construction of the motor, the load it is driving, and the driver attached to motor. Figure 3.2 is showing the min, max and average step response position accuracy measured over one revolution.

Fig.3.2. Step response position accuracy measured over one revolution. The arrows indicate the peak to peak static step accuracy at a point where the motor shaft has settled.
3.10.3 MICRO-STEP ACCURACY

Micro-stepping is often used to position the shaft of a stepper motor between the full step positions. As illustrated in the figure 3.3, the shaft of a motor doesn't always follow the ideal micro-step position dictated by the drive system. In this case, 256:1 microstepping was applied to a 1.8° motor over one electrical cycle (4 full steps). The ideal shaft position (due to drive system pulses), the actual shaft position were measured simultaneously at every micro-step command for 1024 steps. Micro-stepping accuracy is determined by the construction of the motor and the accuracy of the driver. Micro-stepping is rarely more accurate than 1/10th of a step.

![Fig.3.3 Ideal shaft position, actual shaft position and shaft position error.](image)

3.10.4 TORQUE VS. SPEED CHARACTERISTICS

Rotational speed is proportional to stepping rate and inversely proportional to the number of steps per revolution. Mechanical characteristic consists of two areas limited by pull-in torque and pull-out torque curves also known as slewing curves (Fig.3.4.).

![Fig.3.4. Torque vs. speed characteristics](image)
The pull-in torque depends on the total inertia and is a measure at which the motor can start without losing steps. The pull-out torque represents the allowable load that can be supplied at the motor maximum stepping rate after it has reached its speed. The area between the curves (slew range) is the unstable range of the motor, in which the motor may tend to fall out of step and stop.

3.11 COMPUTER AIDED TESTING OF STEPPER MOTOR

In general laboratory stand consists of two main parts „motor” and „computer” one (Fig.3.5.).

![Schematic presentation of laboratory stand.](image)

„Motor” part is the set of several elements like tested step motor itself together with electronic commutator and voltage supply unit, special mechanical arrangement that allows locked rotor test, friction type braking torque device, gear that works together with revolution to pulse converter and set of mirrors that enables to measure step angle.

„Motor” side and „computer” side are electrically separated with the help of optocoupler devices.

![“Motor” part of laboratory stand.](image)

1 – Base, 2 – Step motor EDS-18 model, 3 – Clutch, 4 – Device to evaluate static torque, 5 – Step number encoder,
6 – Baking torque device (friction type), 7 – Gear, 8 – Revolution/Pulse Converter (RPC)
“Computer” part is composed of PC type computer, universal counters card and appropriate software. Five 16-bit counters are used to generate pulse frequency, to control number of steps that are to be performed, to measure real time, and to measure forward and backward angular shaft displacement (Fig.3.7.).

![Diagram of Universal Counters Arrangement](image)

Fig.3.7. Universal counters arrangement.

### 3.11.1 SHAFT POSITION MEASUREMENT

There are several position sensing techniques available. The most widely used is adaptation of revolution to pulse converter (RPC). It has some advantages over other position sensing techniques like for example potentiometer technique. RPC requires only a counter to pass data to the computer and since in the set of counters that is used to create pulse generator there are some spare, implementation of RPC needs no additional equipment. The highest number of pulses per revolution the highest shaft position measurement accuracy can be achieved. But there is technical limitation concerning number of pulses per revolution available in RPC. In this case RPC of 5000 pulses per revolution is used. To achieve measurement error less than 0.2% in respect to one step the additional gear box between motor shaft and RPC shaft is added.

### 3.11.2 SOFTWARE

In order to assure wide range of measurement system flexibility both counter drivers and data acquisition software were written with help of Turbo Pascal/C and Matlab languages respectively. This allow for easy extension of measurement and data processing procedures. Especially Matlab offers wide range of easy to use data visualisation procedures. Its Graphical User’s Interface appears to be an ideal tool to create users friendly operation system.
Fig. 3.8. Graphical User’s Interface window.
Angular shaft displacement measurement is presented.

Fig. 3.9. Graphical User’s Interface window.
Angular shaft velocity measurement is presented.
3.11.3 ANALYSIS OF MEASUREMENT ACCURACY

During the experiment stepper motor performs two steps with the 2 Hz step rate. Software sampling loop is organized in such a way that 1000 data samples are collected during about 1.5 sec. Due to existing gear $3^\circ$ rotation of stepper motor shaft (equivalent to single step) is transformed into:

$$3^\circ \times \frac{D}{d} = 3^\circ \times \frac{30 \text{ [cm]}}{1.952 \text{ [cm]}} = 46.511^\circ,$$

of RPC shaft rotation,

were: $D$ – diameter of stepper motor shaft gear wheel;
$d$ – diameter of RCP shaft gear wheel.

That means that change of the shaft angular position after performing one step is coded by number:

$$E\left(\frac{N_p}{360^\circ}\right) \times 46.511^\circ = E\left(\frac{5000}{360^\circ}\right) \times 46.511^\circ = 646,$$

of pulses,

where: $N_p$ – number of pulses generated by RPC during one full rotation of its shaft.

So resolution with which the shaft actual position can be determined is equal to:

$$\frac{3^\circ}{646} = 0.00464^\circ,$$
and accuracy is equal to 0.1548 % in respect to one step.

Counter 3, the real time counter increases its value every $10^{-5}$ s. (input frequency = 100 kHz). The real time evaluation accuracy is:

$$(10^{-5} \text{ [s]} \times 100\%) / 1 \text{ [s]} = 0.001\%,$$

(3.12)

in respect to two registered steps performed with step rate of 2 Hz. Accuracy of measurement of mean sampling period Is as follow:

$$(10^{-5} \text{ [s]} \times 100\%) / (1 \text{ [s]} / N_s) = (10^{-5} \text{ [s]} \times 100\%) / (1 \text{ [s]} / 1000) = 1\%,$$

(3.13)

where: $N_s$ – number of data samples.

**SELECTED REFERENCES**

4. T.J.E. Miller, Switched reluctance motors and their control, OUP, 1993