EQUIVALENT CIRCUIT OF TWO-WINDING TRANSFORMER

Full representation of the real transformer

Features: Real transformer with two windings (primary & secondary) of different numbers of turns (usually). Sinusoidal voltage supply. The windings are characterised by their resistances $R_1$ and $R_2$. The magnetic core with power losses and with the main (mutual) flux $\Phi$ and leakage fluxes $\Phi_{l1}$ and $\Phi_{l2}$. Primary current $I_1$ and secondary current $I_2$ flow.

Electromotive forces due to mutual flux are induced accordingly:

$$E_1 = 4.44N_1f\Phi$$
$$E_2 = 4.44N_2f\Phi$$

Emfs due to leakage fluxes:

$$E_{x1} = 4.44N_1f\Phi_{l1}$$
$$E_{x2} = 4.44N_2f\Phi_{l2}$$

Leakage fluxes are directly proportional to currents $I_1$ and $I_2$ and emfs corresponding to leakage fluxes can be represented by the voltage drops at leakage reactances $X_1$ and $X_2$.

Representation of the transformer by semi-ideal transformer (with no leakage fluxes and zero-resistance winding)
Assume that we recalculate secondary emf into new value that will be called secondary emf referred to primary

\[ E_2' = E_2 \frac{N_1}{N_2} = 4.44N_2 f \Phi \frac{N_1}{N_2} = 4.44N_1 f \Phi = E_1 \]

and in the same manner secondary output voltage referred to primary

\[ U_2' = U_2 \frac{N_1}{N_2} \]

Now the beginnings of primary and secondary windings have the same potentials \((E_2' = E_1)\)

Referred secondary circuit has to represent the real quantities of energy conversion. It means that the values of power and losses cannot be changed in the new model of transformer. Therefore:

**secondary current referred to primary:**

\[ U_2' I_2' = U_2 I_2 \quad \Rightarrow \quad I_2' = I_2 \frac{N_2}{N_1} \]

**secondary resistance referred to primary:**

\[ R_2'(I_2')^2 = R_2 I_2^2 \quad \Rightarrow \quad R_2' = R_2 \left( \frac{N_1}{N_2} \right)^2 \]

and in the same manner secondary reactance referred to primary:

\[ X_2' = X_2 \left( \frac{N_1}{N_2} \right)^2 \]

The points having the same potentials \((E_1 = E_2')\) can be connected without any current flow. We have now only one electrical circuit without electrical separation between primary and secondary circuits.

How we can represent (by means of electrical circuit) the semi-ideal magnetic core with two identical emfs induced in the windings and with power losses in magnetic circuit?
EQUIVALENT (ELECTRICAL) CIRCUIT OF TWO-WINDING TRANSFORMER

Parallel branch of this T-type circuit represents the magnetic core of the transformer:

iron-core resistance \( R_{Fe} \) - the resistance having the value corresponding to power loss in the magnetic circuit, according to relation:

\[
\Delta P_o = \Delta P_{Fe} = \Delta P_h + \Delta P_e = R_{Fe} I_{ow}^2
\]

magnetizing reactance \( X_f \) - the reactance of primary circuit corresponding to mutual flux and representing primary emf due to relation:

\[
E_1 = I_f X_f \quad \text{or} \quad E_1 = jX_f I_f = E_2^* \]

where:

\[
I_o = I_f + I_{ow} \quad \text{or} \quad I_o = \sqrt{I_f^2 + I_{ow}^2}
\]

Phasor diagram for transformer equivalent circuit:

and voltage balance equations:

\[
U_1 = R_1 I_1 + jX_1 I_1 + E_1 = I_1 (R_1 + jX_1) + I_2 (R_2' + jX_2') + U_2'
\]
NO-LOAD OPERATION AND NO-LOAD TEST

At no-load state the secondary circuit of the transformer is opened:
\[ I_2 = 0; \quad I_1 = I_o \]
Active power supplied to the transformer at no-load state:
\[ P_{1o} = \Delta P_{Cuo} + \Delta P_{Fe} = R_1 I_o^2 + \Delta P_{Fe} \]
No-load loss (power loss not depending on the value of current):
\[ \Delta P_o = \Delta P_{Fe} = P_{1o} - R_1 I_o^2 \]
No-load test of the transformer is easy to be made in laboratory or even in a substation:

\[ RI \]
\[ A \]
\[ W \]
\[ V_1 \]
\[ V_2 \]

Measuring circuit diagram

During test the supplying voltage is varied from 0 up to \( U_N \) (sometimes more). The following quantities are measured:
\[ U_1; U_2; I_o; P_{1o} \]

Appropriate characteristics are drawn in the function of primary voltage:

\[ Basic \ no-load \ characteristics \]

From the no-load test results we can determine some parameters of the transformer being tested or its equivalent circuit:
\[ \cos \varphi = \frac{P_{1o}}{U_1 I_o} \quad I_{ow} = I_o \cos \varphi \quad I_f = I_o \sin \varphi \]
\[ R_{fe} = \frac{\Delta P_o}{I_{ow}^2} \approx \frac{U_1^2}{\Delta P_o} \quad X_f = \frac{U_1}{I_f} \]

Rated no-load current (i.e. the value of no-load current at rated voltage) expressed in p.u. or in percent
\[ I_{oN} = \frac{I_o}{I_N} \quad I_{oN\%} = \frac{I_o}{I_N} \times 100\% \]
is rather of small value: from a few % in large power transformers to 20-30% in small transformers.

\[ RI \] – induction regulator (voltage source of variable output),
\[ A \] – ammeter,
\[ V \] – voltmeter,
\[ W \] – wattmeter

Observe the course of \( I_o \) characteristic! Doesn’t it look as magnetizing curve?
Try to explain why?
Observe also very interesting course of no-load power factor curve. How it can be explain from the physical meaning point of view?
Notice that no-load power factor value for rated voltage is very low!

The value of \( \Delta P_{FeN} \) (iron core loss at nominal voltage) is always given in ratings of transformers or at the nameplates.
MAGNETIZING CURRENT HARMONICS

Assumption: transformer (at no-load = electromagnetic coil) is supplied with the voltage of sinusoidal waveform. This voltage must be balanced by sinusoidal emf induced in the primary winding, hence the flux (flux density) has to be of sine waveform.

Determination of no-load current waveform

- \( i_0 \) is a sum of active \( (i_{0w}) \) and reactive \( (i_r – magnetising current) \) components.
- \( i_f \) is of non-sinusoidal waveform: \( i_f = i_{f1} + i_{f3} + i_{f5} + i_{f7} + \ldots \) (odd harmonics, the 3\(^{rd}\) one is the most important from higher order harmonics).

Conclusion: In the transformer supplied from sinusoidal voltage source the waveform of magnetizing current absorbed from the same source tends to be disturbed from the sinusoidal pattern – at least 3\(^{rd}\) harmonic is required!

Question: What can happen when the source cannot provide higher harmonics of current to flow?
In such situation the flux (flux density) waveform contains 3\(^{rd}\) and other odd harmonics, therefore emf \( e \) waveform is affected by higher harmonics (first of all by 3\(^{rd}\) harmonic). See next figure:

\[ B(t) - \text{sinusoidal} \]
\[ \Phi = f(i) \text{ and } B = f(i) \text{ must be determined from the characteristic of electromagnetic circuit with-} \]
\[ \text{satisfaction of the core and-} \]
\[ \text{hysteresis phenomena taken into consideration.} \]

\[ C_1C_2 = ON_1 – \text{from rising-up part of hysteresis loop;} \]
\[ E_1E_2 = OL_1 – \text{from falling-down part of hysteresis loop} \]
At short circuit the secondary terminals of transformer are short-circuited. $U_2$ is equal to zero and $I_2$ and $I_1$ can be very high (depending on voltage). They are much higher than $I_o$, therefore, after assuming $I_o \approx 0$ the equivalent circuit can be simplified:

We define the following parameters:

- short-circuit resistance and reactance
  
  \[ R_k = R_1 + R_2' \quad \text{and} \quad X_k = X_1 + X_2' \]

- short-circuit impedance (sometimes called equivalent impedance of transformer)
  
  \[ Z_k = R_k + jX_k \quad \text{and} \quad Z_k = \sqrt{R_k^2 + X_k^2} \quad (Z_k = \frac{U}{I} \quad \text{in short-circuited transf.}) \]

Phasor diagram for short-circuited transformer:

- can be supplemented by $R_k$, $X_k$, $Z_k$ triangle (short-circuit impedance) triangle.

In laboratory or during field tests the short-circuit tests are very often performed. For the voltage varied from zero up to the value providing the flow of current not too much higher than $I_N$ they are measured: $P_1$, $I_1$, $\cos\varphi$ and their characteristics drawn:

Due to $I_o \approx 0$ the parallel branch of the circuit can be neglected. Only series resistances and reactances appear in equivalent circuit.

As $X_k$ corresponds to leakage fluxes, the saturation effect doesn't affect its value and we can assume that $X_k$ is constant, independently on current or voltage. It means also that $Z_k = \text{const.}$
For $I_2 = I_N$ we determine and specify:

$U = U_k$ – short-circuit voltage (having two components: $U_{Rk}$ and $U_{Xk}$);
$P_1 = P_k$ – short-circuit power;
$\cos \phi = \cos \phi_k$ – short-circuit power factor.

**Short-circuit voltage** is the value of supplying voltage in short-circuited transformer that provides a flow of rated currents ($I_2$ and $I_1$). Its value is always shown in ratings of transformers or at the nameplates (usually in p.u. or in %).

Application of per unit (or relative) values:

$$I_r = \frac{I}{I_N}; \quad U_r = \frac{U}{U_N}; \quad P_r = \frac{P}{S_N}; \quad Z_r = \frac{Z}{Z_N} \quad \text{where} \quad Z_N = \frac{U_N}{I_N}$$

yields:

$$U_{kr} = \frac{U_k}{U_N} = \frac{I_N}{U_N} = \frac{Z_k}{Z_N} = Z_{kr}$$

Typical percent value of short-circuit voltage is of the following level:
- several % for small and medium power transformers,
- 10 to 20% for large power transformers (hundreds of MVA).

Short-circuit power, i.e. the power absorbed by short-circuited transformer is totally “lost” in the transformer (the output power $P_2 = 0$). The iron losses in such transformer are much smaller when compared to copper loss and can be neglected. Hence, short-circuit power (absorbed by the transformer with rated current) can be calculated as follows:

$$P_k = I_N^2 R_k = I_{1N}^2 + (I_{2N})^2 R_2 = \Delta P_{CuN}$$

The value of $\Delta P_{CuN}$ (rated copper loss, or – in other words – copper loss at rated load) is always shown in ratings of transformers or at the nameplates.

It is easy to prove that:

$$\Delta P_{CuN\%} = U_{Rk\%} \left( \frac{\Delta P_{CuN\%}}{S_N} \cdot 100\% \right)$$

and other interesting relation concerning the value of short-circuit current flowing in short-circuited transformer supplied with rated voltage (rated short-circuit current):

$$I_{krN} = I_{krN} = \frac{U_N}{Z_k} = \frac{1}{Z_k} = \frac{1}{U_{kr}}$$

During test the value of $\cos \phi_k$ can be measured directly by power factor meter or calculated from other values readings:

$$\cos \phi = \frac{P}{S} = \frac{P}{UI}$$

Try to prove it!

Practical meaning of this relation is as follows: imagine the transformer having short-circuit voltage=10%. In such transformer the short-circuit current at nominal supplying voltage is $10 \times I_N$. 

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Try to prove it!
TRANSFORMER UNDER LOAD

For simplified consideration of the transformer operation under load we can apply its simplified equivalent circuit.

Simplification of equivalent circuit (EC) leads to some error – much lower when \( I_o \) is smaller in comparison to \( I_1 \) or \( I_2 \) currents. Hence, we can use such EC for discussing power transformers fully loaded.

Using such model we can consider the problem of output voltage variation for different value and character of the load current. Let us consider the following problem: transformer is supplied with constant primary voltage \( U_1 \) and the loading current \( I_2 \) varies. The character of load current is given by its phase angle \( \phi_2 \) varying at closed interval \([-90^\circ, +90^\circ]\).

For \( I_2'=I=0 \) the output voltage \( U_{2o}'=U_1 \). For \( I>0 \) and some \( \phi_2 \)

we observe the voltage drop \( \Delta U \) (difference in voltage values):

\[
\Delta U = U_1 - U_2 = U_{2o}' - U_2 = \overline{CA} \approx \overline{CB} = IR_1 \cos \phi_2 + IX_\alpha \sin \phi_2
\]

Assuming \( U_1=U_{1N} \) and expressing \( \Delta U \) in per units

\[
\Delta U_r = \frac{U_{2oN}' - U_2}{U_{2oN}} = \frac{U_{2oN'} - U_{2o}}{U_{2oN}} = \frac{I}{I_N} (R_1 \cos \phi_2 + X_\alpha \sin \phi_2)
\]

and for \( I=I_N \)

\[
\Delta U_r = R_1 \cos \phi_2 + X_\alpha \sin \phi_2 = U_{1N} \cos \phi_2 + U_{3N} \sin \phi_2
\]

and final expression for the value of output voltage

\[
U_2 = U_{2oN} \left( 1 - \frac{I}{I_N} \Delta U_{rN} \right)
\]

In similar manner, for any value of supply voltage \( U_1 \) and corresponding value of \( U_{2o} \) we can calculate the output voltage

\[
U_2 = U_{2oN} \left( 1 - \frac{U_{1N}}{U_1} \frac{I}{I_N} \Delta U_{rN} \right) \quad \text{or} \quad U_2 = U_{2oN} \left( 1 - \frac{S}{S_N} \left( \frac{U_{1N}}{U_1} \right)^2 \Delta U_{rN} \right)
\]

At this figure the voltage drops triangle is drawn not to the scale. The hypotenuse DC in case of \( I=I_N \) corresponds to the value of \( U_k \) and can be few percent of \( U_1 \).
The function $\Delta U = f(\cos \phi_2)$ for $I = I_N$ is shown in following figure.

It is seen that $\max(\Delta U) = U_k$ occurs at $\cos \phi_2 = \cos \phi_k$ i.e. for very special value being a known parameter of the transformer (short-circuit power factor).

Some interesting practical observations can be concluded:

1. in the transformer fully loaded ($I = I_N$ – it is very much required not to overload the transformer above its rated current) the voltage drop at internal impedances of the transformer can be maximum $U_k$ (or in p.u. – $U_{kr}$ or $U_{k\%}$). For example in the transformer having $U_{k\%} = 18\%$ the maximum voltage drop (assuming $I \leq I_N$) can be $18\%$, i.e.
   $$\max \left( \frac{U_{2o} - U_2}{U_{2o}} \times 100\% \right) = 18\%$$
   from where it follows that minimum output voltage could be of the value of $82\%$ of $U_{2o}$ and it would appear for $\cos \phi_2 = \cos \phi_k$

2. For some capacitive loads (leading character of the current) the voltage drop can be of negative value, what means the output voltage at such load can be higher than at no load.

Summarising above we can conclude that in the transformer having high value of short-circuit voltage one can expect correspondingly large variation of output voltage with varying load.

Imagine your transformer having rated voltages $15000/220$ V. These voltages are voltage values at no-load. Let your transformer operates as step-down and let it to be supplied with $15 \, \text{kV}$ and fully loaded. Would you like to have your home appliances to be supplied with the voltage $0.82 \times 220 = 180$ V?