3-PHASE INDUCTION MOTORS

EXAMPLES OF MOTOR OPERATION

At the base of equivalent circuit showing one phase of multi-phase motor

we can consider different possibilities of motor operation, depending on its speed $n$, slip $s$ and, therefore, the values of:

resistance $R_2 \cdot \frac{1-s}{s}$$

and the power $I_2' \cdot R_2 \cdot \frac{1-s}{s}$

a) $[n = n_1]$ ($s = 0$) - ideal no-load operation (zero torque)

$R_2 \cdot \frac{1-s}{s} = \infty$ (electrical equivalent circuit is opened – no-load).

Output power shown by means of this resistance is zero ($I_2' = 0$).

b) $[n = 0]$ ($s = 1$) (the shaft is blocked – is locked – locked rotor state)

$R_2 \cdot \frac{1-s}{s} = 0$ (electrical eq. circuit is short – circuited:
short-circuit state of the motor).

Output power shown by means of this resistance is zero.
**Operation of induction machine with** \(0 < n < n_1\) (motor operation)

Sankey’s diagram – energy (power) balance diagram:

\[
P_1(P_{in}) \quad P_1 = m_1 I_1 U_0 \cos \varphi
\]

\[
\Delta P_{Cu1} = m_1 I_1^2 R_1 \text{ - stator copper loss} \quad (m_1 - \text{number of phases})
\]

\[
\Delta P_{Fe} \text{ - stator core loss (can be determined from the no-load test)}
\]

The remainder: \(P_e = P_1 - \Delta P_{Cu1} - \Delta P_{Fe}\)

is called “**electromechanical power**” or “**ideal power**”. Physically it is power transferred from the stator to rotor by means of magnetic field (through the air gap). It is a power delivered to the rotor!

From the equivalent circuit:

\[
P_e = m_1 I_2^2 \frac{R_s}{s} = m_1 I_2^2 R_2' + m_1 I_2^2 R_2' \frac{1-s}{s}
\]

rotor copper loss \(\Delta P_{Cu2}\) “mechanical power” \(P_m\)

or “rotor developed power”

\[
\Delta P_{Cu2} = sP_e \quad P_m = P_e - \Delta P_{Cu2}
\]

From the first relation: \(\rightarrow\) the value of slip \(s\) should be as small as possible to limit the value of rotor copper loss. In practice \(s\) is at the range of several %, therefore \(f_2 = sf_1\) is very small and rotor iron (core) loss is small and can be neglected.

After subtraction mechanical loss (power loss due to friction & windage of rotating rotor) from \(P_m\) we get:

\[
P_m - \Delta P_m = P_2 \quad (\text{or } P_{out} \text{ or } P) - \text{shaft output power}
\]

Efficiency of energy (power) conversion:

\[
\eta = \frac{P_2}{P_1} = \frac{P_{out}}{P_{in}}
\]

For further consideration let’s take the mechanical power which corresponds to the power appearing across the resistance \(R_2' \frac{1-s}{s}\). This power is connected with the torque- **electromechanical torque** – driving the rotor with the speed \(n\) (or \(\Omega\) in \(\text{rad s}^{-1}\)):

\[
P_m = m_1 I_2^2 R_2' \frac{1-s}{s}
\]

Electromechanical torque

\[
T_e = \frac{P_m}{\Omega} = \frac{60 m_1}{2 \pi (1-s)} I_2^2 R_2' \frac{1-s}{s} \quad \text{in [N-m] or [Nm]}
\]

Rotor core loss is not even shown at Sankey’s diagram.
From simplified equivalent circuit:

\[
\begin{align*}
I_1 & \quad X_1 \quad X_2 \\
\text{U}_1 & \quad I_2' \quad R_2'/s
\end{align*}
\]

With two following simplifications:

a) \(R_1 \ll \frac{R_2}{s}\) - and can be neglected,

b) parallel branch is neglected,

\[
I_2' = \frac{U_1}{\sqrt{\left(X_1 + X_2\right)^2 + \left(\frac{R_2}{s}\right)^2}}
\]

Substitution \(I_2'\) to \(T_e\) relation yields:

\[
T_e = \frac{60m_1}{2\pi n_1} \cdot \frac{R'_2 U_1^2}{s(X_1 + X_2)^2 + \left(\frac{R_2}{s}\right)^2}
\]

In general \(T_e = f(s, U_1, f, R_2)\)

The torque developed in the machine is proportional to the voltage squared.

Testing of the function \(T_e = f(s)\) for \(U_1, f, R_2 = \text{const}\) yields the following results:

**torque–slip curve**

There are two critical points

\[
T_{\text{max}} = \pm \frac{60m_1}{2\pi n_1} \frac{U_1^2}{2(X_1 + X_2)}
\]

**breakdown torque** – maximum torque that can be developed in IM for given voltage \(U_1\) and frequency \(f\).

\(T_{\text{max}}\) doesn't depend upon \(R_2\)!

\[
s_{\text{max}} = \pm \frac{R_2'}{X_1 + X_2}
\]

**breakdown slip** \(s_{\text{max}} \sim R_2\)!

The same result in other coordinates

**torque–speed curve**

Kloss formula

\[
T_{\text{max}} = \frac{2}{s + \frac{s_{\text{max}}}{s_{\text{max}}}}
\]

\(T_s\) - **starting torque**.

Usually \(s_{\text{max}}\) is at the range of about 10%.
STABILITY OF THE MOTOR OPERATION

The motor operates with constant speed when torque balance is zero:

\[ T_e - T_{load} = 0 \]

From two possible points A & B only point B corresponds to stable operation.

Stable region of \( T-n \) curve

Stable region is for \( 0 < s < s_{\text{max}} \)

\( N \) - point of nominal operation

(\( T_N, n_N \) - nominal torque, nominal speed).

Adequate surplus of stability is required. Usually minimum value of torque ratio required by users is 2:

\[ \frac{T_{\text{max}}}{T_N} \geq 2 \]

stability margin

For motor driving the load having fan-type torque-speed characteristic it can happen that stable operation occurs for very low speed:

Such operation is very dangerous for normal construction motor:

\[ n \approx \frac{1}{2} n_1 \quad s \approx 0.5 \]

\[ \Delta P_{Cu2} \approx 0.5 P_e \]

what means:

- very high rotor copper loss!,
- very low efficiency!.
NUMERICAL EXAMPLE

Consider a 3-phase induction machine (motor) of rating:

\[
P_N = 75 \text{ kW} \quad \text{rated power (always must be understood as output one)}
\]

\[
U_N = 220/380 \text{ V} \quad \text{rated voltage (for two possible connections)}
\]

\[
\cos \phi_N = 0.85 \quad \text{rated power factor}
\]

\[
\eta_N = 0.92 \quad \text{rated efficiency}
\]

\[
f = 50 \text{ Hz}
\]

\[
n_N = 975 \text{ rev/min} \quad \text{rated speed}
\]

\[
\Delta P_m = 0.5\%P_N \quad \text{mechanical loss at rated speed}
\]

\[
R_1 = 0.033 \Omega \quad \text{stator winding (phase) resistance}
\]

Calculate:
1) Nominal stator current (line) for star and delta connections of stator winding,
2) Apparent nominal power \( S_N \) (power drawn by the stator from the line),
3) Active and reactive power absorbed from the mains for nominal load,
4) Nominal torque and nominal slip,
5) Iron core loss.

1.
\[
I_{INY} = \frac{P_N}{\sqrt{3}U_N \cos \phi_N} = \frac{75 \times 10^3}{\sqrt{3} \times 380 \times 0.92 \times 0.85} = 145.7 \text{ A}
\]

\[
I_{IN\Delta} = \frac{P_N}{\sqrt{3}U_N \eta_N \cos \phi_N} = \frac{75 \times 10^3}{\sqrt{3} \times 220 \times 0.92 \times 0.85} = 251.7 \text{ A}
\]

2. \[
S_N = \frac{P_N}{\cos \phi_N} = \frac{75}{0.92 \times 0.85} = 95.9 \text{ kVA}
\]

3. \[
P_{IN} = \frac{P_N}{\eta_N} = \frac{75}{0.92} = 81.52 \text{ kW}
\]

\[
Q_{IN} = S_N \sin \phi_N = 95.9 \sqrt{1 - 0.85^2} = 50.5 \text{ kvar}
\]

4. \[
T_N = 9.55 \frac{P_N}{n_N} = 9.55 \times \frac{75 \times 10^3}{975} = 734.6 \text{ N} \cdot \text{m}
\]

\[
s_N = \frac{n_1 - n_N}{n_1} = \frac{1000 - 975}{1000} = 0.025 \quad s_N = 2.5\%
\]

5. \[
\Delta P_{CuIN} = 3I_{Nph}^2R_1 = 3 \times 145.7^2 \times 0.033 = 2.102 \text{ kW}
\]

\[
\begin{align*}
P_e &= P_{IN} - \Delta P_{FeN} - \Delta P_{CuIN} \\
P_N &= P_m - \Delta P_m = P_e (1 - s) - \Delta P_m
\end{align*}
\]

from this set of equations the unknown \( \Delta P_{FeN} \) is

\[
\Delta P_{FeN} = P_{IN} - \Delta P_{CuIN} - \frac{P_N + \Delta P_m}{1 - s} = 81.52 - 2.102 - \frac{75(1 + 0.005)}{1 - 0.025} = 2.11 \text{ kW}
\]
**Slip-ring induction machine**

Idea of construction

Example of connections

![Connections of slip-ring machine.](image)

Basic characteristics

![Basic characteristics graph](image)
Squirrel-cage machine (squirrel-cage motor, cage motor)

Different possible profiles of rotor slots and cage bars

Double-cage motor

Deep-bar rotor (deep-bar squirrel-cage induction motor)
STARTING OF INDUCTION MOTORS

Slip-ring motor starting

Cage motor starting

Star-delta starting (most popular for the motors of rated power above 5 kW).
NUMERICAL EXAMPLE

About induction motor of rating as in previous numerical example:

\[ P_N = 75 \text{ kW} \]
\[ U_N = 220/380 \text{ V} \]
\[ \cos \phi_N = 0.85 \]
\[ \eta_N = 0.92 \]
\[ f = 50 \text{ Hz} \]
\[ n_N = 975 \text{ rev/min} \]
\[ \Delta P_m = 0.5\%P_N \]
\[ R_1 = 0.033 \Omega \]

we know that it is a squirrel cage motor having the breakdown torque ratio \( T_{\text{max}}/T_N = T_b/T_N = 2.2 \), starting torque ratio \( T_s/T_N = 1.3 \) and rated starting current ratio \( I_{sN}/I_N = 6 \). Calculate:

1. Values of starting torque and currents for direct switch-on starting (at nominal conditions of supply),
2. Starting torque and current for \( Y/\Delta \) starting.

1. a) for \( Y \) connection (stator connected in \( Y \), \( U=U_N=380 \text{ V} \))

\[ T_{sN} = 1.3 \times T_N = 1.3 \times 734.6 = 954.98 \text{ N}\cdot\text{m} \]
\[ I_{sNY} = 6 \times I_{NY} = 6 \times 145.7 = 874.2 \text{ A} \]

b) for \( \Delta \) connection (stator connected in \( \Delta \), \( U=U_N=220 \text{ V} \))

\[ T_{sN} = 1.3 \times T_N = 1.3 \times 734.6 = 954.98 \text{ N}\cdot\text{m} \]
\[ I_{sN \Delta} = 6 \times I_{N \Delta} = 6 \times 251.7 = 1510.2 \text{ A} \]

2. stator circuit connected in \( Y \) and supplied from the mains \( 3 \times 220 \text{ V} \):

\[ I_{sY} = \frac{1}{\sqrt{3}} \times 6 \times I_{NY} = \frac{6}{\sqrt{3}} \times 145.7 = 504.72 \text{ A} \]

\[ \frac{I_{sY}}{I_{sN \Delta}} = \frac{1}{\sqrt{3}} \times \frac{6 \times I_{NY}}{6 \times \sqrt{3} \times I_{NY}} = \frac{1}{3} \]

\[ T_{sY} = \left( \frac{U}{U_N} \right)^2 T_{sN} = \left( \frac{1}{\sqrt{3}} \right)^2 T_{sN} = \frac{1}{3} T_{sN} = \frac{1}{3} \times 954.98 = 318.33 \text{ N}\cdot\text{m} \]
SPEED CONTROL OF INDUCTION MOTOR

By means of rotor resistance

Speed control by means of rotor resistance.

By means of line voltage

Speed control by means of line voltage.

By means of slip-power recovery

Slip-power recovery.
**SPECIAL APPLICATION OF INDUCTION MACHINE**

**INDUCTION REGULATOR**

By means of frequency

Torque/speed characteristics for variable-frequency control.

![D.C. link inverter diagram](image)

Action of inverter.

Rotor position is controlled by means of worm gear (worm/pinion device).

Controlled voltage (AC) source $0 \div 2V_1$ (laboratory application).